NOTATION

T, characteristic function of a process (temperature, concentration); x, coordinate; δ, δ_1, distance between measurement points; a, β, γ, transfer constants; Fo, $\tilde{\text{Fo}}$, Fourier numbers; t, τ, θ, time.

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THEORY OF A THERMAL DIFFUSION APPARATUS WITH TRANSVERSE FLOWS

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UDC 621.039.341.6

The article discusses a continuous thermal diffusion apparatus in which supply and removal take place at the ends of the separating slit. The dependence of the shift in concentration on the parameters of the apparatus, the properties of the mixture, and the amount of fluid removed is determined.

All known types of thermodiffusion cascades of constant, stepped, or ideal profile are characterized by the fact that the mixture being separated moves through elements forming a given cascade. The scheme for connecting thermodiffusion columns proposed by Jones and Frazier [1, 2] and shown schematically in Fig. 1a is distinguished by the fact that the mixture is pumped outside the separating part of the column. As can be seen from the figure, the mixture being separated is delivered to the top and bottom ends of the outermost columns and moves along the respective ends until it leaves the cascade. A theory of such a cascade proposed in [3] was constructed on simplified model representations applying to the separation of petroleum products. In connection with the latter, the relations obtained here are approximate.

The present work attempts to avoid the above problems and is based on the use of classical theory [4].

A battery of columns (Fig. 1a) may be represented in an idealized variant as a plane column, the top and bottom parts of which contain channels 2 (indicated by the dashed line in Fig. 1b, c) connected with the separating part of the apparatus 1. The apparatus is divided into a series of narrow columns by vertical barriers 3. It is assumed that diffusion along the x axis in these columns may be ignored, which allows us to regard the problem as being unidimensional within each column. The same assumption is made with regard to diffusion in the top and bottom channels, which is fully justified given the fairly high flow rates typical of the chosen operating regime. It is further assumed that the convective flow entering the channels 2 from the region 1 is ideally mixed along the z axis with the flows passing through the channels.

In any vertical cross section of the apparatus being examined, transfer in the case of a binary mixture is determined by the formula

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 41, No. 2, pp. 231-238, August, 1981. Original article submitted June 4, 1980.

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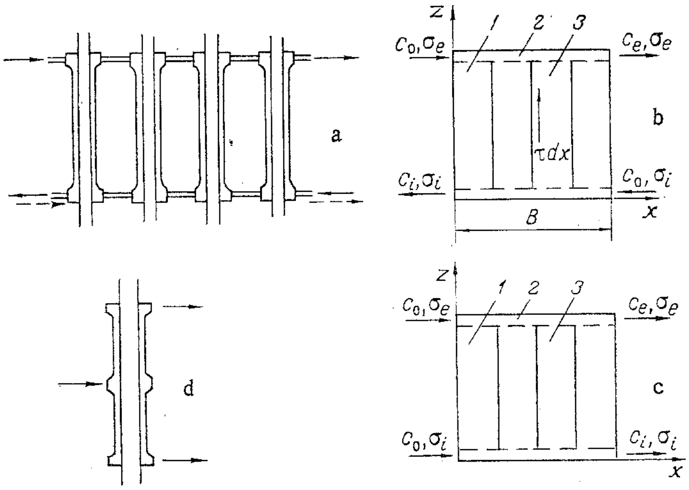

Fig. 1. Column-operating schemes: a) with transverse flows; b, c) idealized model of counterflow and forward flow; d) with central feed.

$$\tau = Hc(1-c) - K\frac{dc}{dz}, \tag{1}$$

which, with introduction of the variable

$$Hz/K = j \tag{2}$$

takes the form

$$\tau = H\left[c(1-c) - \frac{dc}{dy}\right]. \tag{3}$$

To obtain meaningful final results, we will limit ourselves to the case of linear approximation of the nonlinear term

$$c(1-c) \approx a + bc. \tag{4}$$

Since transfer is a constant quantity in the steady state, then, differentiating (3) with respect to y, we obtain

$$\frac{d^2c}{dy^2} - b\frac{dc}{dy} = 0. \tag{5}$$

The solution to (5) is

$$c = A_1(x) + A_2(x)e^{by}, \tag{6}$$

where the coefficients $A_1(x)$, $A_2(x)$ depend on x, i.e., on the location of the vertical cross section being examined. To find these constants, let us examine a section of the column of length dx (see Fig. 1b). The object component $\tau dx/B$ flowing into the top channel from the column is mixed with the flow σ_e, increasing the concentration in the channel by dc, i.e., when $y = y_e$

$$\left.\frac{\tau}{B\sigma_e}dx\right|_{y=y_e} = dc_e. \tag{7}$$

In the bottom channel ($y = 0$), the change in the concentration of the object product

$$\left.\frac{\tau}{B\sigma_i}dx\right|_{y=0} = \mp dc_i, \tag{8}$$

where the top and bottom signs pertain respectively to the cases of forward flow and counterflow. After the quantity τ in Eqs. (7) and (8) is replaced by the value of τ from (3), (7) and (8) are the boundary conditions for finding $A_1(x)$, $A_2(x)$:

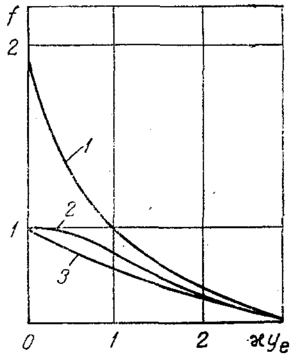

Fig. 2. Dependence of f on the value of $\varkappa y_e$: 1) counterflow; 2) forward flow; 3) central feed.

$$\left.\frac{dc}{dx}\right|_{y=y_e} = \frac{1}{B\varkappa_e}\left(a + bc - \frac{dc}{dy}\right)_{y=y_e},$$
$$\left.\frac{dc}{dx}\right|_{y=0} = \mp\frac{1}{B\varkappa_i}\left(a + bc - \frac{dc}{dy}\right)_{y=0}, \tag{9}$$

where

$$\varkappa = \sigma/H. \tag{10}$$

On the other hand, from (6)

$$\left.\frac{dc}{dx}\right|_{y=y_e} = A_1'(x) + A_2'(x)\,e^{by_e},$$
$$\left.\frac{dc}{dx}\right|_{y=0} = A_1'(x) + A_2'(x). \tag{11}$$

Substituting Eqs. (6) and (11) into (9), we obtain two differential equations for finding the sought quantities:

$$A_1'(x) + A_2'(x)\,e^{by_e} = \frac{1}{B\varkappa_e}[a + bA_1(x)],$$
$$A_1'(x) + A_2'(x) = \mp\frac{1}{B\varkappa_i}[a + bA_1(x)]. \tag{12}$$

Forward Flow. Simultaneous solution of (12) yields

$$A_1(\xi) = C_1 e^{-\varphi_p\xi} - \frac{a}{b},$$
$$A_2(\xi) = C_1\left(\frac{b}{\varkappa_i\varphi_p} - 1\right)e^{-\varphi_p\xi} + C_2, \tag{13}$$

where

$$\xi = \frac{x}{B};\quad \varphi_p = \frac{b}{\varkappa_i}\,\frac{\exp(by_e) + \varkappa_i/\varkappa_e}{\exp(by_e) - 1}, \tag{14}$$

with C_1 and C_2 being integration constants.

Substituting the corresponding values from (13) into (6), we obtain

$$c_e = C_1 e^{-\varphi_p\xi} + \left[C_1\left(\frac{b}{\varkappa_i\varphi_p} - 1\right)e^{-\varphi_p\xi} + C_2\right]e^{by_e} - \frac{a}{b}, \tag{15}$$

$$c_i = C_1\frac{b}{\varkappa_i\varphi_p}\,e^{-\varphi_p\xi} - \frac{a}{b} + C_2. \tag{16}$$

To find the constants C_1 and C_2, we take into consideration that $c_e = c_0$ and $c_i = c_0$ with forward flow at the origin, i.e., when $\xi = 0$. Using these boundary conditions, we obtain the following dependences on the dimensionless coordinate ξ for the change in concentration of the top and bottom flows:

$$c_e = c_0\left[e^{-\varphi_p\xi} + \left(1 + \frac{\varkappa_i}{\varkappa_e}\right)\frac{(1 - e^{-\varphi_p\xi})\, e^{b\xi}}{e^{by_e} + \frac{\varkappa_i}{\varkappa_e}}\right] + \frac{a}{b}(1 - e^{-\varphi_p\xi})\frac{\varkappa_i}{\varkappa_e}\,\frac{e^{by_e} - 1}{e^{by_e} + \frac{\varkappa_i}{\varkappa_e}}, \tag{17}$$

$$c_i = c_0\,\frac{(e^{by_e} - 1)\, e^{-\varphi_p\xi} + \left(1 + \frac{\varkappa_i}{\varkappa_e}\right)}{e^{by_e} + \frac{\varkappa_i}{\varkappa_e}} - \frac{a}{b}\,\frac{(e^{by_e} - 1)(1 - e^{-\varphi_p\xi})}{e^{by_e} + \frac{\varkappa_i}{\varkappa_e}}. \tag{18}$$

Counterflow. In this case, after solving (12) and substituting into (6)

$$c_e = D_1 e^{\varphi_c\xi} + \left[D_1\left(\frac{b}{\varphi_c\varkappa_i} - 1\right) e^{\varphi_c\xi} + D_2\right] e^{by_e} - \frac{a}{b}, \tag{19}$$

$$c_i = D_1\,\frac{b}{\varphi_c\varkappa_i}\, e^{\varphi_c\xi} - \frac{a}{b} + D_2, \tag{20}$$

where

$$\varphi_c = \frac{b}{\varkappa_i}\,\frac{e^{by_e} - \frac{\varkappa_i}{\varkappa_e}}{e^{by_e} - 1}. \tag{21}$$

Since the feed flows are entering from opposite ends of the apparatus, to find the constants D_1, D_2 we should use the condition $c_e|_{\xi=0} = c_0$, $c_i|_{\xi=1} = c_0$, which gives

$$c_e = c_0\,\frac{\frac{\varkappa_i}{\varkappa_e}(e^{by_e} - 1)\, e^{\varphi_c\xi} + e^{by_e}\left(e^{\varphi_c} - \frac{\varkappa_i}{\varkappa_e}\right)}{e^{by_e+\varphi_c} - \frac{\varkappa_i}{\varkappa_e}} - \frac{a}{b}\,\frac{(e^{by_e} - 1)(e^{\varphi_c} - e^{\varphi_c\xi})}{e^{by_e+\varphi_c} - \frac{\varkappa_i}{\varkappa_e}}, \tag{22}$$

$$c_i = c_0\,\frac{(e^{by_e} - 1)\, e^{\varphi_c\xi} + e^{\varphi_c} - \frac{\varkappa_i}{\varkappa_e}}{e^{by_e+\varphi_c} - \frac{\varkappa_i}{\varkappa_e}} - \frac{a}{b}\,\frac{a\,(e^{by_e} - 1)(e^{\varphi_c} - e^{\varphi_c\xi})}{e^{by_e+\varphi_c} - \frac{\varkappa_i}{\varkappa_e}}. \tag{23}$$

The special case corresponding to values of $a = 0$ and $b = 1$ in Eqs. (17), (18), (22), and (23) was examined in [5]. It should be stipulated that all of the formulas presented above are valid only when $\varkappa_i > 0$ and $\varkappa_e > 0$, since only then will condition (9) remain in force.

As can be seen from (14), (17), (18), and (21)-(23), the concentrations at the outlet of the apparatus are functions of the three parameters $\varkappa_i$, $\varkappa_i/\varkappa_e$, and y_e. To simplify analysis of the results obtained, we will examine the case where $b = 0$. In accordance with (4), this case corresponds to the value $c(1 - c) \approx 1/4$.

It can then be shown that with forward flow

$$(c_{ek} - c_{ik})_p = \frac{y_e}{4}(1 - e^{-\varphi_p}),\quad \varphi_p = \frac{1 + \frac{\varkappa_i}{\varkappa_e}}{\varkappa_i y_e}, \tag{24}$$

while with counterflow

$$(c_{ek} - c_{ik})_c = \frac{y_e}{4}\left(1 + \frac{\varkappa_i}{\varkappa_e}\right)\frac{e^{\varphi_c} - 1}{e^{\varphi_c} - \frac{\varkappa_i}{\varkappa_e}},\quad \varphi_c = \frac{1 - \frac{\varkappa_i}{\varkappa_e}}{\varkappa_i y_e}. \tag{25}$$

To evaluate the efficiency of each variant of moving the transverse flows with respect to one another, it is expedient to use the quantity

$$f = \frac{4\,(c_{ek} - c_{ik})}{y_e}, \tag{26}$$

where $y_e/4$ is the shift in concentration in the column being operated without removal of fluid [4]. The results of such a comparison, when $\varkappa_e = \varkappa_i$, are shown in Fig. 2. It is apparent from the figure that, in the region of values $\varkappa y_e < 1.6$, the counterflow is not only greater than the forward flow, but it makes it possible to obtain an increment in concentration which is almost double the increment obtainable by the column in the no-removal regime. A further comparison was made for a centrally fed column (Fig. 1d) in which

$$f = \frac{2}{\varkappa y_e}\left[1 - \exp\left(-\frac{\varkappa y_e}{2}\right)\right]. \tag{27}$$

Curve 3 in Fig. 2 shows that the central-feed scheme is about as efficient as forward flow (Fig. 1c).

The formulas presented above were analyzed in connection with schemes b and c in Fig. 1, corresponding to operation of the apparatus on the principle of ideal displacement. In practice, when the apparatus consists of a series of separate columns (Fig. 1a), each having top and bottom parts containing small tanks, the contents of which may be mixed with the flow entering from the separating part, then there will not be a concentration gradient in each of the tanks along the direction of motion of the flows σ_e and σ_i. Examining this scheme, we assume that the perimeter of all of the columns B are the same in magnitude and we limit ourselves to the case where $c(1 - c) \approx \text{const} = a$. A change in concentration in the top of the n-th column is equal to the difference between the concentration c_{en} at which the flow σ_e leaves the n-th column and the concentration c_{en-1} at which it enters from the (n − 1)-st column, i.e., $c_{en} - c_{en-1}$. This increment in concentration is due to the entry from the separating part of the flow defined by (3), i.e., with allowance for the chosen approximation

$$a - \left(\frac{dc_n}{dy}\right)_{y=y_e} = \varkappa_e' (c_{e,n} - c_{en-1}). \tag{28}$$

If the flow σ_i in the bottom of the apparatus is sent in the same direction as the flow σ_e (forward flow), then, by analogy with (28),

$$a - \left(\frac{dc}{dy}\right)_{y=0} = -\varkappa_i' (c_{in} - c_{in-1}). \tag{29}$$

In the case of counterflow, instead of (29) we have

$$a - \left(\frac{dc_n}{dy}\right)_{y=0} = \varkappa_i' (c_{in+1} - c_{in}). \tag{30}$$

Since the flow (3) is constant in any cross section of the column in the steady state, $(dc_n/dy)_{y=y_e} = (dc_n/dy)_{y=0} = dc_n/dy$. Then, integrating (28)-(30) with respect to y and keeping in mind that $c_n|_{y=y_e} = c_{en}$, $c_n|_{y=0} = c_{in}$,

$$c_{en} - c_{in} = ay_e - \varkappa_e' y_e (c_{en} - c_{en-1}), \tag{31}$$

$$c_{en} - c_{in} = ay_e - \varkappa_i' y_e (c_{in-1} - c_{in}), \tag{32}$$

$$c_{en} - c_{in} = ay_e - \varkappa_i' y_e (c_{in+1} - c_{in}). \tag{33}$$

The single quote mark in the above equations indicates that the value of H in (10) is taken for a single column. Having determined $c_{en} - c_{en-1}$ from (31) and $c_{in-1} - c_{in}$ from (32) and having added the left and right sides of the resulting expressions, we find the relationship between the difference in the concentrations at the inlet and outlet of the n-th column:

$$c_{en} - c_{in} = ay_e \frac{u}{y_e + u} + \frac{y_e}{y_e + u} (c_{en-1} - c_{in-1}), \tag{34}$$

where

$$u = \frac{1}{\varkappa_e'} + \frac{1}{\varkappa_i'}. \tag{35}$$

It is obvious that for the first column $c_{en-1} = c_{in-1} = c_0$, so that

$$c_{e1} - c_{i1} = ay_e \frac{u}{y_e + u}. \tag{36}$$

Substituting (36) into (34), we obtain an expression for $c_{e2} - c_{i2}$. Using this expression, we find $c_{e3} - c_{i3}$, etc. As a result, for the difference $c_{eN} - c_{iN}$, where N is the total number of columns in the apparatus, we obtain a geometric progression. Finally, for the sought difference in concentrations, we find

$$c_{eN} - c_{iN} = ay_e\left[1 - \left(\frac{y_e}{u + y_e}\right)^N\right]. \tag{37}$$

With $N \to \infty$, the fraction in the brackets vanishes and we obtain the result for a column from which no fluid is being removed. The values of c_{eN} and c_{iN} can be found from the balance equation

$$\sigma_e(c_{eN} - c_0) = \sigma_i(c_0 - c_{iN}). \tag{38}$$

In the case of counterflow from (32) and (33), instead of (34)

$$c_{en} - c_{in} = ay_e\frac{u}{u + y_e} + \frac{y_e}{u + y_e}(c_{en-1} + c_{in+1}). \tag{39}$$

Using the balance equation

$$\sigma_e(c_{en} - c_{en-1}) = \sigma_i(c_0 - c_{in}) \tag{40}$$

and substituting the value of c_{in} determined from this equation into (39), after simple transformations we obtain

$$\frac{c_{eN} - c_0 + \frac{\varkappa_e'}{\varkappa_i'}ay_e}{1 + \varkappa_i' y_e} + \frac{\varkappa_i'}{\varkappa_e'}\frac{1 + \varkappa_e' y_e}{1 + \varkappa_i' y_e}(c_{en} - c_0) = (c_{en-1} - c_0). \tag{41}$$

With $n = 1$ and $c_{en-1} = c_0$, the right side of the last equation vanishes and gives the following relation for the first column

$$\frac{c_{eN} - c_0 + \frac{\varkappa_e'}{\varkappa_i'}ay_e}{1 + \varkappa_i' y_e} + \frac{\varkappa_i'}{\varkappa_e'}\frac{1 + \varkappa_e' y_e}{1 + \varkappa_i' y_e}(c_{e1} - c_0) = 0, \tag{42}$$

while for the second column

$$\frac{c_{eN} - c_0 + \frac{\varkappa_e'}{\varkappa_i'}ay_e}{1 + \varkappa_i' y_e} + \frac{\varkappa_i'}{\varkappa_e'}\frac{1 + \varkappa_e' y_e}{1 + \varkappa_i' y_e}(c_{e2} - c_0) = c_{e1} - c_0. \tag{43}$$

The expressions for columns 3, 4,...N will be similar. Inserting the value of $c_{e1} - c_0$ from (43) into (42) and then successively replacing $c_{e2} - c_0$ from Eq. (41) for the third column, etc., we obtain a geometric progression, the sum of which, after the appropriate transformations, gives the following for N columns

$$c_{eN} - c_0 = ay_e\frac{1 - [\varkappa_i'(1 + \varkappa_e' y_e)/\varkappa_e'(1 + \varkappa_i' y_e)]^N}{(\varkappa_e'/\varkappa_i') - [\varkappa_i'(1 + \varkappa_e' y_e)/\varkappa_e'(1 + \varkappa_i' y_e)]^N}. \tag{44}$$

With $\varkappa_e' = \varkappa_i' = \varkappa'$, from (44) we have

$$c_{eN} - c_0 = ay_e\frac{N}{1 + \varkappa' y_e + N}. \tag{45}$$

The increase in the concentration of the object product with an increase in the number of columns will vary, depending on the relationship between $\varkappa_i'$ and $\varkappa_e'$. This becomes apparent if we set $N \to \infty$:

$$(c_{eN} - c_0)_{\varkappa_i' < \varkappa_e'} = ay_e\frac{\varkappa_i'}{\varkappa_e'}, \quad (c_{eN} - c_0)_{\varkappa_i' > \varkappa_e'} = ay_e, \tag{46}$$

i.e., in the second case, the increase in concentration at the positive end of the apparatus will be higher. This has been confirmed by experimental data [6]. It is interesting to note that in the approximation being examined ($b = 0$), Eq. (22) — with allowance for (21) — takes the following form at $\xi = 1$

$$c_{ek}-c_0=ay_e\frac{\varkappa_i}{\varkappa_e}\frac{\exp\left[\left(1-\frac{\varkappa_i}{\varkappa_e}\right)\Big/\varkappa_i y_e\right]-1}{\exp\left[\left(1-\frac{\varkappa_i}{\varkappa_e}\right)\Big/\varkappa_i y_e\right]-\frac{\varkappa_i}{\varkappa_e}}, \tag{47}$$

and in the case of large B degenerates into the relation

$$(c_{ek}-c_0)_{\substack{B\to\infty\\ \varkappa_i<\varkappa_e}}=ay_e\frac{\varkappa_i}{\varkappa_e},\quad (c_{ek}-c_0)_{\substack{B\to\infty\\ \varkappa_i>\varkappa_e}}=ay_e, \tag{48}$$

Comparison of Eq. (48) with (46) shows that operation of the apparatus according to the ideal-displacement scheme is more efficient at $\sigma_i > \sigma_e$ and that (46) and (48) give the same result only when $\varkappa_i = \varkappa_e$. It can be shown that when large amounts of fluid are removed and $y_e \approx 1$, nearly the same results as in the scheme in Fig. 1c are achieved with as low a number of columns $N \approx 5$ if they are connected in accordance with the countercurrent scheme in Fig. 1a.

NOTATION

B, column perimeter; c, concentration; D, diffusion coefficient; $H = \alpha_T \rho^2 g \beta \delta^3 (\Delta T)^2 B / 6! \eta \bar{T}$; $K = g^2 \rho^3 \beta^2 \delta^7 (\Delta T)^2 B / 9! \eta^2 D$; L, column height; y, dimensionless coordinate; α_T, thermodiffusion constant; β, coefficient of cubical expansion; δ, gap in the column; $\varkappa$, dimensionless removal rate; η, viscosity; λ, thermal conductivity; σ, removal rate. Indices: c, countercurrent; e, positive end of column; i, negative; n, column number; p, forward flow.

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